Final Performance Report
for Optimization Problems in Multisensor and
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13. SUPPLEMENTARY NOTES

14. ABSTRACT
The objective of this research program is to develop optimization algorithms that solve key problems in multiple target tracking and sensor data fusion. The central problem in multiple target tracking is the data association problem of partitioning sensor reports into tracks and false alarms. New classes of data association problems have been formulated and initial algorithms developed to address cluster tracking, merged measurements, and even sensor resource management in the form of ``group-assignments." In a different direction, an efficient k-best algorithm has been developed to approximate the uncertainty in data association, which is critical for discrimination or combat identification. Statistical Monte Carlo methods are also applicable and are still under investigation. Bias estimation algorithms using known data association such as truth objects and targets of opportunity have been developed. Bias estimation in which data association is unknown is difficult due to the nonconvex and mixed integer nature of the mathematical formulation. Exact and approximate algorithms have been developed and successfully applied to system tracking. As a prerequisite to the development of multiple target tracking approaches to space surveillance, consistent measures of uncertainty for initial orbit determination and the propagation of the uncertainty over time have been developed.

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1   OBJECTIVES

The objectives of this research program are to develop optimization algorithms that solve key problems in multiple target tracking and sensor data fusion. While these optimization problems are motivated by multiple target tracking applications, the class of problems and corresponding algorithms are abstracted to a more mathematical setting so that the resulting research developments have a wider applicability.

The first class of these, the data association problem, is the central problem in multiple target tracking. This is manifest in partitioning measurements into tracks and false alarms, and also in associating tracks from multiple sources. For measurement-to-measurement or measurement-to-track association, the multidimensional assignment problem governs the association process. Lagrangian relaxation are combined with an exact method such as A*-search or branch and bound, which if run to completion, would produce an optimal answer. (Optimal solutions are required to run off-line in simulations to verify that the approximate algorithm is solving the association problem to within the noise level in the problem.) While the basic algorithms for constructing a solution are now mature for single assignments and work exceptionally well, new problems and demands require research into new formulations and algorithms supported by mathematical theory.

The first of these arise from the need to perform group-formation tracking for ground targets, group-cluster tracking and pixel-cluster tracking for IR sensors, and to process merged measurements for in (narrowband) radar. Thus, an objective is to formulate a class of new problems within the framework of the multidimensional assignment problem, namely the "group-assignment" problems, which have strong similarities to multiple dimensional combinatorial auctions. These problems reduce to the traditional one-to-one multidimensional assignment when the groups do not overlap. In addition, these same group-assignment problems also appear to have much broader application to new problems arising in network management, procurement, or resource scheduling under the name "combinatorial auctions."

In addition to this new class of combinatorial optimization problems, an important new area for multiple target tracking is that of supplying some measure of uncertainty in the data association process. Here is the reason. Most tracking systems already supply a covariance matrix to represent the uncertainty in the track state due to measurement noise and mismodeling of the dynamics of the object. This uncertainty does not represent the data association uncertainty, i.e., the measurement-to-track or track-to-track assignment. Almost all target identification, combat identification, or discrimination algorithms assume perfect data association and are not robust to misassociation; however, tracking systems cannot produce perfect data association given the stochastic nature of the problem. Thus, in order to properly process kinematic and feature data in the identification, combat identification, and discrimination process, an assessment of the association process is required. This capability will be required by all future tracking systems. While we have developed an efficient k-best algorithm to approximate this uncertainty, Markov Chain Monte Carlo methods are being investigated as well.

A third component of this research program is the treatment of sensor biases and

navigation errors, which is a prerequisite to multi-sensor tracking and fusion. These errors contain a deterministic component (e.g., a mean) as well as a purely stochastic component. One can estimate a deterministic component using a nonlinear maximum a posteriori (MAP) estimation formulation, which is posed as a weighted nonlinear least squares problem when the errors are assumed to be Gaussian. The least squares problem requires training data that can be either from known data association such as targets of opportunity in which the target's dynamic state must also be estimated or certain or uncertain truth data which does not need to be estimated.

A fourth component is the problem of joint bias estimation and data association using the target themselves in which data association is unknown. This leads to a class of nonconvex mixed integer nonlinear programming problems. Once considered too difficult for practical applications, this problem is taking center stage due to the need to correct biases in track states transmitted over a network where access to the fine details of the sensor measurement process is not available. This problem is central to processing UAV track states, space surveillance, system level tracking, and missile defense.

A fifth component of this work has been the initial investigations into initial orbit determination, the uncertainty associated with the estimates, and the propagation of the uncertainty over time. *A separate report is attached to this one and is summarized below.*

While the above four classes of problems remain a focus, Numerica continues the work on network-centric tracking that was initially started with AFOSR funding and has now transitioned to a Phase II SBIR at AFRL/SNAT and a Phase II SBIR with Department of the Army with a transition path to the SIAP Joint Program Office.

1.1 STATUS OF EFFORT

The status of the current research program is summarized in this section. The individual topics are addressed in the following subsections. Of these, the most significant part of the effort in the last year has been the development of a general approach to the joint bias estimation and data association discussed below.

1.1.1 The Group-Assignment Problem

Over the last fifteen years, the multidimensional assignment problem in which one assigns each measurement or track at most once has generally been accepted as the correct formulation of the central data association problem for multiple hypothesis tracking (MHT) and is now the industry standard. (This does not mean that all systems have adopted it; many US military systems use very old tracking technologies.) Improvements in the existing algorithms and the development of new algorithms for this problem will remain a fundamental research problem for decades to come.

While the current research program has pursued improvements in these algorithms, a focus has been to investigate new classes of data association problems arising from a need to extend current capabilities to tracking large numbers of closely spaced objects, tracking in dense clutter on the one hand, breaking up merged measurements over what a radar

or IR sensor can normally resolve. These represent three different classes of problems: a) one in which measurements are merged or clustered to improve run-time performance, b) individual object tracking in which each object is tracked, and c) measurements or clusters are decomposed to better track subclusters or individual objects. In a) and c), one can use clustering methods to formulate the data association problem. We now give a brief description of these problems.

The idea behind problem classes a) and c) is to use clustering methods to either group measurements together or to break up clusters or merged measurements. If the correct number of clusters is known a priori, then the problem could be posed as a normal assignment problem that allows multi-assignment to split or merge clusters. Determining the number of clusters in cluster tracking or in clustering methods in general is a *key problem*. While there are many suggested methods for determining the number of clusters given just one look at the data, most of these are not robust in tracking where one makes multiple looks at the data (frames of data) as it evolves in time. *To take advantage of this feature of the tracking problem, we have proposed multiple clustering of the data on each frame and then deciding on the correct number on a particular frame by examining many frames at once.*

The corresponding mathematical formulation of the problem can be stated as follows. Let P and Q denote two lists of objects and let $\mathcal{P} = \{P_i\}_{i \in I}$ and $\mathcal{Q} = \{Q_j\}_{j \in J}$ denote collections of subsets of P and Q, respectively. These collections represent all the subclusters obtained from multiple clusterings of the data. A general formulation of the cluster tracking multi-assignment problem allows the multi-assignment between groups P_i of $\mathcal{P}$ and Q_j of $\mathcal{Q}$ directly and then adds the set packing as additional constraints.

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{(i,j) \in \mathcal{A}} c_{ij} x_{ij}, \\
\text{Subject To} \quad & \sum_{j \in \mathcal{A}(i)} x_{ij} \leq m_i \quad (i \in I), \\
& \sum_{i \in \mathcal{B}(j)} x_{ij} \leq n_j \quad (j \in J), \\
& x_{i_1 j_1} + x_{i_2 j_2} \leq 1 \text{ for all } (i_1, j_1) \text{ and } (i_2, j_2) \in \mathcal{A} \\
& \qquad \text{for which } i_1 \neq i_2 \text{ and } P_{i_1} \cap P_{i_2} \neq \emptyset \\
& \qquad \text{or } j_1 \neq j_2 \text{ and } Q_{j_1} \cap Q_{j_2} \neq \emptyset, \\
& x_{ij} \in \{0, 1\}.
\end{aligned} \tag{1}
$$

The constraint (1) is the constraint on the (hard) set packing requirement in clustering. It essentially says that a measurement cannot be in two different clusters in the final assignment.

More extensive discussions of this formulation as well as extensions to higher dimensions are presented in the work of Gadaleta and Poore [3, 13, 4, 12, 11]. The formulation of the cluster assignment problem is particularly well-suited to a Lagrangian relaxation algorithm in that the set packing constraint can be Lagrangian relaxed to the base prob-

lem consisting of either the usual one-to-one assignment problem or the multi-assignment problem. The nonsmooth optimization of the resulting problem is relatively straightforward. The final step in restoring the set packing constraint is that which remains.

Clustering Methods. Clustering Methods is a very large subject indeed, and there are so very many clustering methods [15]. Since the error in measurements is normally Gaussian, we have used a Gaussian sum approximation and the expectation maximization (EM) algorithm to determine the means and covariances as well as the cheaper K-Means algorithm. A new approximation due to Maybeck and Williams [16] seems to be a very promising approach that can replace the EM algorithm. This has yet to be investigated at Numerica. (Note that the track state updates are clustered in the work of Maybeck and Williams while the measurements themselves are clustered in this work.)

The Status. The following gives a brief description of the major tasks of this proposed effort. Though not mentioned explicitly earlier, each of these formulations has been tried on small problems where the optimal solution can be obtained by explicit enumeration, and the results have been shown to be very effective in cluster tracking. The number of problems listed here exceeded our budget and future work will need to focus on these problems to further develop the algorithms; however, they are all essential pieces to solve the next generation of data association problems.

1. **Two-Dimensional Multi-Assignment Solver.** Numerica has proposed the extension of the JV algorithm to the multi-assignment case since it is the fastest known algorithm for the one-to-one assignment problem. An algorithm has been completed that incorporates the enumeration of the K-best solutions based on Murty's method.

2. **Two-Dimensional Group-Assignment Solver (With and Without Multi-Assignments).** The easiest problem in this group is the one without multi-assignments. Numerica uses this formulation in tracking using UAV platforms.

1.1.2 Assignment Ambiguity Assessment and the Status.

The initial approach to assessing ambiguity in data association was to compute the K-Best solutions up to some maximum value of K or to some point at which the objective function value drops below some threshold relative to the optimal solution. These costs are then used to estimate (optimistically) the probabilities of association. While this has worked well for sparse to medium density problems, a more robust algorithm is needed for the medium to dense problems and one that augments the K-Best solutions. Work in this direction is proceeding using K-Best and an exploration technique to sample parts of the solution space to ensure that declarations of high probability of association are indeed correct. Numerica has investigated statistical Monte Carlo Methods, e.g., importance sampling and Markov Chain Monte Carlo methods. The results will be published in future work.

1.1.3   Bias Estimation.

Sensor bias and platform navigation errors represent systematic unmodeled measurement and navigation errors, i.e., they cannot be averaged out. In single sensor environment, biases can lead to optimistic and diverging filters. In multi-sensor environments bias errors can lead to data misassociation (due to optimistic covariances or large residual biases) and erratic stereo multiple motion model performance. (In the worst case, N sensors can produce N times the true number of tracks.) There are various classifications of the biases such as absolute vs. relative and sensor biases vs. navigation errors. In relative bias estimation, some of the biases are determined relative to others; the corresponding estimation techniques generally assume these to be known a priori.

A key requirement for bias estimation is the need for training data. There are many proposals in the literature including targets of opportunity in which data association is known by the various sensors and truth objects, e.g., precision reports based on GPS. In general, the state-of-the-art is to use targets with *known* data association spread over the field of regard to increase the number of biases that are observable from the data. Such data may not always be present or is very difficult to obtain. In the quest for a rich set of training data, one could turn to the targets themselves, but here the problem is that the data association is *unknown*. This has been considered to be too difficult a problem in that it results in a nonconvex mixed integer nonlinear programming problem. Thus, one can delineate the bias estimation problems into two classes: a) those with known data association using either truth objects or objects of opportunity and b) those with unknown data association. We now address both of these problems.

The methods being investigated herein use numerical linear algebra techniques in the estimation procedure (e.g., singular value decomposition) to determine the relative biases a posteriori as part of the solution.

Bias Estimation Using Known Data Association.   In funding from both AFOSR, AFRL/SNAT, and Numerica internal funding, Numerica has undertaken systematic treatment of biases based on "batch ML/MAP estimation with process noise," which in many cases reduces to a nonlinear least squares problem. (The correct data association is assumed.) The singular value decomposition is used to determine the bias roll-ups and the observable biases. The initial results [10] and later more careful analysis of Herman and Poore [5] using truth objects and the work of Kragel, Herman, Danford and Poore [7] lead to performance of the multisensor tracking comparable to performance without biases. These works are based on a nonlinear least squares formulation and the the singular value decomposition to determine the observable and unobservable biases.

Here is a brief technical description of the problem. In the absence of bias, a least-squares estimate for the target state based on noisy measurements is obtained by mini-

mizing the objective function (see Jazwinski[6], Sage and Melsa[14] or Poore[9] et al.)

$$\frac{1}{2}\sum_{i=1}^{N_t}\left(\sum_{(s,k)\in M_i}\|z_k^{(s)}-h_k^{(s)}(x_k^{(s)})\|_{(R_k^{(s)})^{-1}}^2\right. \tag{2}$$

$$\left.+\sum_{k=2}^{T_i}\|x_k^{(i)}-f_{k-1}(x_{k-1}^{(i)})\|_{Q_k^{-1}}^2+\|x_1^{(i)}-f_0(x_0^{(i)})\|_{(F_1 P_0 F_1^T+Q_1)^{-1}}\right),$$

where T_i is the number of (unique) time updates for target i, F_k is the Jacobian of f_k, P_0 is the covariance of the state prior, and s and k denote the sensor index and time index, respectively. The first term in (2), the *sensor component*, embodies the influence of the observations on the estimated target track, while the second term, the *process noise component*, enforces model fidelity. The third term, the *state prior component*, incorporates information based on previous estimates. Note that the Mahalanobis distance is used in (2) rather than the Euclidian distance. This strongly influences the result of the estimation procedure since the relative influence of each of the components is determined by the relationship between the matrices $R_k^{(s)}$, $Q_k^{(s)}$ and P_0.

Adding bias estimation to (2) is now straightforward. We replace the measurement model in the sensor component with the modified model of (3) and append *bias process noise* and *bias prior components* similar to the corresponding state components. This yields the objective function

$$\Psi(\chi)=\frac{1}{2}\sum_{i=1}^{N_t}\left(\sum_{(s,k)\in M_i}\|d_k\left(z_k^{(s)},b_k^{(M)(s)}\right)-h_k^{(s)}\left(x_k^{(s)},b_k^{(T)(s)}\right)\|_{(R_k^{(s)})^{-1}}^2\right. \tag{3}$$

$$\left.+\sum_{k=2}^{T_i}\|x_k^{(i)}-f_{k-1}(x_{k-1}^{(i)})\|_{Q_k^{-1}}^2+\|x_1^{(i)}-f_1(x_0^{(i)})\|_{(F_1 P_0 F_1^T+Q_1)^{-1}}\right) \tag{4}$$

$$+\frac{1}{2}\sum_{s=1}^{N_s}\left(\sum_{k=2}^{T_s}\|b_k^{(s)}-f_k^b(b_{k-1}^{(s)})\|_{(Q_k^b)^{-1}}^2+\|b_1^{(s)}-f_0^b(b_0^{(s)})\|_{(F_1^b P_0^b(F_1^b)^T+Q_1^b)^{-1}}^2\right), \tag{5}$$

where N_s denotes the number of sensors, T_s is the number of (unique) update times from sensor s and

$$\chi=\left\{x_k^{(i)},b_k^{(T)(s)},b_k^{(M)(s)}\right\},\quad (s,k)\in M_i,\ i=1,\dots,N_t, \tag{6}$$

is the vector of unknown states and biases for all targets and sensors in the current estimation window.

Once sensor biases and navigation errors are estimated and the biases and errors corrected, one is still left with the stochastic part of the biases called residual biases. We have dealt with these via an extension of the Schmidt-Kalman filter to the nonlinear problem and multiple interacting filter models [8]. The combination extensively improves tracking performance.

Status. This work is now complete.

Bias Estimation Using Targets with Unknown Data Association. The simplest of these problems occurs in trying to match track states from two different sources. This is a key problem in associating and fusing track states from UAVs or in *space surveillance* where the sensor reports are track states rather than measurements. The problem can be posed as the problem of determining a displacement d and an assignment x that optimize the the mixed integer nonlinear programming problem

$$\text{Minimize}_{(x,d)}\ c_r(d) + \sum_{(i,j)\in A} c_{ij}(d)x_{ij}$$

$$\text{Subject To: } \sum_{j\in A(i)} x_{ij} \le 1 \quad (i = 1, ..., m), \tag{7}$$

$$\sum_{i\in B(j)} x_{ij} \le 1 \quad (j = 1, ..., n),$$

$$x_{ij} \in \{0, 1\}$$

$$\tag{8}$$

where A is the set of feasible arcs, $A(i) = \{j \,|\, (i, j) \in A\}$ and $B(j) = \{i \,|\, (i, j) \in A\}$ and

$$c_r(d) = \frac{1}{2}d^T R^{-1} d$$

$$c_{ij}(d) = \frac{1}{2}(\hat{x}_i + d - \hat{y}_j)^T Z_{ij}^{-1}(\hat{x}_i + d - \hat{y}_j) \tag{9}$$

$$+ \frac{1}{2} \ln\left[\beta_T^2 \det\left(2\pi Z_{ij}\right)\right] + \gamma_{ij} \text{ for } i \ne 0 \text{ and } j \ne 0, \tag{10}$$

$$Z_{ij} = P_{ii} - S_{ij} - S_{ji} + Q_{jj} \tag{11}$$

where β_T denotes the target density, $\gamma_{ij} = \ln\left((1 - P_d^1)(1 - P_d^2)\right)$, and P_d^k, $k \in \{1, 2\}$ is the probability of sensor k observing a particular target.

In addition, there is the *multidimensional assignment* version of this problem [1]. (Note that for each assignment x, there is a unique local minimum d.) The costs $c_{ij}(d)$ can be general functions of d, but the algorithms other than the estimation algorithms for d given an assignment x remain pretty much unchanged. The first class of these algorithms are based on combined use of heuristics and branch-and-bound algorithms or A*-search. This has been a major achievement of the program over the last year with preliminary work being reported by Danford, Kragel, and Poore [1, 2].

Status. The algorithms [1, 2] for the two dimensional problem are now complete, but will need to be coded in an implementation language such as C++. We have developed four classes of algorithms: (a) optimal algorithm based on branch and bound, (b) an optimal algorithm based on A*-search, (c) a heuristic that is very efficient in runtime and is near optimal, and (d) an anytime algorithm. A formulation for the multidimensional problem is complete and algorithms have been planned, but not yet implemented. This work will be submitted for additional publications in the future.

1.1.4 Network-Centric Multiple Frame Assignments.

An objective in this research program is the development of distributed multiple frame data association algorithm that is comparable in quality to that of centralized tracking while managing communication loading and achieving a consistent or single integrated air picture. This work was initially funded by AFOSR; however, due to its importance, AFRL/SNAT and SIAP Joint Program Office (SIAP JPO) both funded Phase II SBIRs. This work is ongoing with strong potentials for transition to SIAP JPO.

1.1.5 Generation and Propagation of Track States and Their Uncertainty in Space Surveillance.

The objective in this research component is the generation of the initial orbit, its uncertainty, and the propagation of this uncertainty over time. This investigation is a prerequisite to the development of an MHT tracker for space surveillance.

Status. An attached report goes into much more detail. This work has just begun and we hope to continue in a new AFOSR contract.

1.2 ACCOMPLISHMENTS/NEW FINDINGS

The central problem in any surveillance system is the data association problem of partitioning observations into tracks and false alarms. Over the last fifteen years and with support from AFOSR, a new approach has been developed based on the use of multidimensional assignment problem formulation and Lagrangian relaxation algorithms. (This approach is often called multiple frame assignments or MFA for short.) Four U. S. patents have now been issued for this work. What is more, based on this new technology, Lockheed Martin of Owego, NY won the best of Breed Tracking Contest for the next upgrade to AWACS held at Hanscom AFB in Boston in 1996, and it has been chosen as the tracking system for the Navy's new multipurpose helicopter under the LAMPS program. Based on much of this work, Numerica has now developed a system level tracker for the Missile Defense Agency that should be deployed in 2008.

The research performed here is fundamental to future needs in tracking and surveillance over the next fifteen to twenty years.

2 PERSONNEL SUPPORTED

a. PI: Aubrey B. Poore

b. Colleagues at Numerica: Eric Kuo, Scott Danford, and Scott Lundberg.

c. Fritz Obermeyer, MS in Mathematics, Colorado State University, 2005, Currently PhD student at Carnegie Mellon University, Pittsburgh, PA.

3 PUBLICATIONS

a. F. H. Obermeyer and A. B. Poore, A Bayesian Network Tracking Database, in Oliver E Drummond, editor, 2004 SPIE Conference Proceedings, Signal and Data Processing of Small Targets, SPIE, August 2004.

b. S. Gadaleta and A. B. Poore and Sean Roberts and B. J. Slocumb, Multiple Hypothesis Clustering Multiple Frame Assignment Tracking, In Oliver E. Drummond, editor, Signal and Data Processing of Small Targets, SPIE Vol. 5428, August, 2004, 294-307.

c. A. B. Poore and S. Gadaleta, The Group Assignment Problem Arising in Multiple Target Tracking, in D. Grundel and R. Murphey and P. Pardalos, Theory and Algorithms for Cooperative Systems, World Scientific, pages 423–449, 2004.

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e. Roman Novosolev, Aubrey B Poore, and Brian J. Suchomel, Network Centric MFA Tracking for Air Defense: A Tribute to Oliver E Drummond, 2005.

f. Sabino Gadaleta, Shawn Herman, Scott Miller, Fritz Obermeyer, Benjamin Slocumb, Aubrey B Poore, and Mark Levedahl, Short-term Ambiguity Assessment to Augment Tracking Data Association Information, Eighth International Conference on Information Fusion, July, 2005, Philadelphia, PA.

g. R. Novoselov and S. Herman and S. Gadeleta and A. Poore, Mitigating the effects of residual biases with Schmidt-Kalman filtering, Eighth International Conference on Information Fusion, July, 2005, Philadelphia, PA.

h. Aubrey B Poore, Complexity Reduction in MFA/MHT Tracking, in Oliver E Drummond, editor, 2005 SPIE Conference on Small Targets, Volume 5913, August, 2005.

i. Allison Floyd, Sabino Gadaleta, Dan Macumber, Aubrey B Poore, Integration of IR Sensor Clutter Rejection Techniques with Pixel Cluster Frame Manipulation, in Oliver E Drummond, editor, 2005 SPIE Conference on Small Targets, Volume 5913, August, 2005.

j. R. Novoselov and S. Gadeleta and A. Poore, Network Centric MFA Tracking Architecture Based on Soft-Level Data Association, in Oliver E Drummond, editor, 2005 SPIE Conference on Small Targets, Volume 5913, August, 2005.

k. Daniel L Macumber, Sabino Gadaleta, Allison Floyd, and Aubrey B Poore, Hierarchical Closely-Spaced Object (CSO) Resolution for IR Sensor Surveillance, in Oliver E Drummond, editor, 2005 SPIE Conference on Small Targets, Volume 5913, August, 2005.

l. Shawn M. Herman and Aubrey B Poore, Nonlinear Least-squares Estimation for Sensor and Navigation Biases, in Oliver E. Drummond, Proceedings of the 2006 SPIE Conference on Signal and Data Processing of Small Targets. Vol 6236, May 2006.

m. Benjamin J Slocumb and Aubrey B Poore, Complexity reduction in MHT/MFA tracking: Part II, hierarchical implementation and simulation results, in Oliver E. Drummond, Ed., Proceedings of the 2006 SPIE Conference on Signal and Data Processing of Small Targets. Vol 6236, April 2006.

n. Aubrey B Poore, Sabino M Gadaleta and Benjamin J Slocumb, Multiple Hypothesis Correlation in Track-to-Track Fusion Management, in Gautam Appa, Leonidas Pitsoulis, and H. Paul Williams, Handbook on Modelling for Discrete Optimization, Springer International Sequence, Chapter 12, pp 341-372, Springer Verlag, New York, 2006.

o. Aubrey B Poore and Sabino Gadaleta, Some Assignment Problems Arising from Multiple Target Tracking, Mathematical and Computer Modelling, Elsevier, Volume 43, pp 1074-1091, 2006.

p. Shawn M. Herman and Aubrey B Poore, Nonlinear Least-squares Estimation for Sensor and Navigation Biases, in Oliver E. Drummond, Proceedings of the 2006 SPIE Conference on Signal and Data Processing of Small Targets. Vol 6236, April 2006.

q. Scott Danford, Bret Kragel, Aubrey Poore, Joint MAP Bias Estimation and Data Association: Algorithms, Proceedings of SPIE Conference on Small Targets, Oliver E Drummond, Ed., Vol 6699, August, 2007.

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s. Bret Kragel, Shawn Herman, Scott Danford, and Aubrey Poore, Bias Estimation Using Targets of Opportunity, Proceedings of SPIE Conference on Small Targets, Oliver E Drummond, Ed., Vol 6699, August, 2007.

t. Randy Paffenroth, Roman Noveselov, Marcio Teixeira, Stephanie Chan, and Aubrey Poore, Mitigation of Biases Using the Schmidt-Kalman Filter, Proceedings of SPIE Conference on Small Targets, Oliver E Drummond, Ed., Vol 6699, August, 2007.

4   INTERACTIONS/TRANSITIONS

a. **Participation/presentations at meetings, conferences, seminars, etc.**

   i. Aubrey B Poore, Multiple Hypothesis Correlation, Project Hercules Review, MITLL (December 2005 and November 2006)

ii. Scott Danford, Bret Kragel, and Aubrey B Poore, Joint MAP Bias Estimation and Data Association: Algorithms and Simulations, SPIE Conference on Signal and Data Processing of Small Targets, July 2007.

iii. Scott Danford, Bret Kragel, and Aubrey B Poore, Joint MAP Bias Estimation and Data Association: Algorithms, SPIE Conference on Signal and Data Processing of Small Targets, July 2007.

iv. Bret Kragel, Shawn Herman, Scott Danford, and Aubrey Poore, Bias Estimation Using Targets of Opportunity, SPIE Conference on Signal and Data Processing of Small Targets, July 2007.

v. Randy Paffenroth, Roman Noveselov, Marcio Teixeira, Stephanie Chan, and Aubrey Poore, Mitigation of Biases Using the Schmidt-Kalman Filter, SPIE Conference on Signal and Data Processing of Small Targets, July 2007.

vi. Optimization Problems in Multiple Target Tracking, AFOSR Workshop, June 2007, Washington, DC.

v. Sensor Resource Management, SIAP Joint Program Office, June 2007.

b. **Consultative and Advisory Functions**

Dr. Poore serves as a Subject Matter Expert on tracking and combat identification for the SIAP Joint Program Office (formerly JSSEO) (August, 2003 - Present).

c. **Transitions**

i. **Transition: Numerica**

Numerica Corporation, a Colorado Corporation, is a small business in Fort Collins, CO that is engaged in basic research, software development, and engineering services, especially in surveillance. Aubrey B. Poore is President. In January of 1999, Numerica completed negotiations with Colorado State University Research Foundation (CSURF) to take out an exclusive license on the U. S. Patents, software, and tracking technology developed by Aubrey Poore at Colorado State University (CSU) (past, present, and future) for the purpose of licensing the tracking technology to industry.

ii. **Transition: SBIRS High, an Air Force Program**

Numerica participated in a risk reduction effort in support of the Air Force Program SBIRS High. The result of this participation was a demonstration that the tracking requirements for SBIRS High could indeed be met.

iii. **Transition: MDNTB**

Numerica transitioned its multi-assignment algorithm and software to the Missile Defense National Team B for use in the ballistic launch event association (BLEA) algorithm that will be deployed in 2006.

iv. **Transition: AFRL, WPAFB**

Numerica transitioned its assignment solver and simulation environment for urban surveillance to AFRL/SNAT during spring of 2007.

iv. **Transition: SBIRS Low (Now STSS)**

Numerica transitioned its MFA tracking system to Spectrum Astro of Gilbert, AZ (now General Dynamics) and to Northrop Grumman Corporation of El Segundo, CA for the SBIRS Low program (now STSS).

v. **Transition: Project Hercules, MDA/BC and MDA's MDNTB**

In a rapid and advanced development program funded by MDA for Project Hercules, Numerica is developing a state-of-the-art tracking system for radar based tracking of missiles for both national and theater missile defense. While SBIRS Low is for satellite based IR sensors, this program focuses on radar based systems located on the ground, on the sea, and in the air. As part of this effort, Numerica has developed and transitioned the track processing threads for MDA's C2BMC to MDA's National Team B. The subject algorithms are being coded for deployment in 2008.

5 NEW DISCOVERIES, INVENTIONS, OR PATENTS

Patents listed below to emanate from the basic research supported by AFOSR. Essentially an entirely new approach to multiple hypothesis tracking (MHT) has been developed.

a. **NEW DISCOVERIES/INVENTIONS:**

The inventions as embodied in the U.S. Patent below contains various approaches to data association problems based on linear programming relaxations and contains a description of the variable depth sliding window.

b. **PATENTS ISSUED for AFOSR SPONSORED RESEARCH**

Aubrey B. Poore, Jr., Method and System for Tracking Multiple Regional Objects by Multi-Dimensional Relaxation, U.S. Patent Number 6,404,380 B2, Issued 11 June 11 2002. (Assignee: Colorado State University Research Foundation, Fort Collins, CO)

Thomas N. Barker, Joseph A. Persichetti, Aubrey B. Poore, Jr., and Nenad Rijavec, Method and System for Tracking Multiple Regional Objects, US Patent Number 5406289, issued 11 April 1995. (Assignees: IBM, Owego, NY; Colorado State University Research Foundation, Fort Collins, CO)

Aubrey B. Poore, Jr., Method and System for Tracking Multiple Regional Objects by Multi-Dimensional Relaxation, CIP, US Patent Number 5537119, issued on 16

July 1996. (Assignee: Colorado State University Research Foundation, Fort Collins, CO)

Aubrey B. Poore, Jr., Method and System for Tracking Multiple Regional Objects by Multi-Dimensional Relaxation, CIP, U. S. Patent Number 5,959,574, issued on September 28, 1999. (Assignee: Colorado State University Research Foundation, Fort Collins, CO)

6 HONORS/AWARDS

a. Name of Award: **The 2004 Colorado State University Research Foundation Technology Transfer Award** This award is given yearly to one of the faculty at Colorado State University for achievements in transitions of technology to industry. A complete description is given at http://www.csurf.org/researcher.html.

Year Received: **2004**

Honor/Award Recipient(s): Aubrey B. Poore

Awarding Organization: Colorado State University Research Foundation

b. Name of Award: **Colorado State University Alumni Association Distinguished Faculty Award**. (The award is given to a current Colorado State University faculty member, not necessarily a Colorado State University graduate, who has demonstrated excellence in teaching, research, and service.)

Year Received: **1999**

Honor/Award Recipient(s): Aubrey B. Poore

Awarding Organization: Colorado State University Alumni Association

c. Name of Award: **The Burlington Northern Faculty Achievement Award for Research and Graduate Education**

Year Received: 1990. (This was the eleventh such award at Colorado State University for graduate education and teaching.)

Honor/Award Recipient(s): Aubrey B. Poore

Awarding Organization: Colorado State University

d. Aubrey Poore continues to server as Associate Editor of *Computational Optimization and Applications*.

14

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Generation and Propagation of Track States and Their Uncertainty in Space Surveillance: An Extended Report for

Final Performance Report
for Optimization Problems in Multisensor and Multitarget Tracking
AFOSR Grant Number FA9550-04-1-0222

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February 25, 2008

1   Introduction

Space surveillance is the component of space situational awareness (SSA) focused on the detection of space objects and the use of multi-source data to track and identify space objects. Currently, it can take weeks to establish correct orbits on objects of interest, due predominantly to the manual processes for correlation of observations on an object. This manual correlation process is particularly fragile when events such as collisions or the recent Chinese ASAT[2] test create large numbers of new observable objects. In order to ensure that the space catalog can be updated in a timely fashion when large numbers of new objects arise, we require modern multiple target tracking methods.

Multiple target tracking requires an accurate accounting of uncertainty in the measurements, biases, and track states for both the initiation of tracks (initial orbit determination) on newly observed objects, and for the propagation of the state estimates on those objects over time. Currently, the space catalog does not incorporate state vector uncertainty in its published Two-Line Element (TLE) sets. Newer proposed data structures for transmitting information on an object's state, such as ISO 22644, Orbit Data Messages [3], may provide an interface for communicating uncertainty data on orbit states. While significant work has gone into developing mechanisms for high-accuracy orbit propagation given an accurate initial orbit state, less focus has been placed on the combined problems of developing an accurate description of the uncertainty in the initial orbit determination, and of propagating the uncertainty along with the state vector. For example, for a newly initiated orbit, inaccuracies in the initial orbit determination can substantially impact the object location days or even hours later. The only way to address this inaccuracy is to develop an uncertainty measure (e.g., a covariance matrix) of the new orbit that correctly reflects the error in the state vector estimate. The metric that we use for describing the correctness of the covariance matrices is "covariance consistency." One of the goals of our effort has been to demonstrate that consistent covariances can be achieved for the initial orbit determination and that they can be maintained when propagated, at least for a period of time measured in days. The achievement of consistent covariance matrices is a prerequisite to the development of an MHT tracker for space surveillance.

Generation of consistent covariances on initial orbit determination requires a consider-analysis or Schmidt filter to account for residual biases or ill-conditioning in the nonlinear transformations between state and measurement space. Propagation of uncertainty over time is also challenging, in that propagation of the linearized uncertainty in a Cartesian coordinate frame will not preserve consistency. Instead, covariance propagation requires use of some form of the classic orbital elements. The work herein focuses on measurements from radar sensors, but the general approach with appropriate modifications can also be used for other sensors (such as optical telescopes).

Section 2 discusses the details of our radar model and the target motion. Section 3 presents the algorithmic approaches to achieving consistent covariances for initial orbit determination and for the propagation of covariances. Section 4 shows the performance of these methods in simulation.

2   Problem Statement

Given a sequence of radar measurements emanating from the same object, our goals are (a) to establish an initial orbit, (b) to determine a consistent covariance matrix for that object, and (c) to propagate the covariance or uncertainty over an extended period of time so that the uncertainty remains consistent. Given the limited nature of this effort, our models are simplified; however, we believe the general methods can be extended to more general force models and different sensors.

2.1   Uncertainty in Radar Measurements

Errors in radar measurements (range, azimuth, and bearing) are significantly greater in the angle components of the measurement than in range. This discrepancy leads to an error ellipsoid that looks rather pancake-shaped in Cartesian coordinates. Figure 1 illustrates this confidence region.
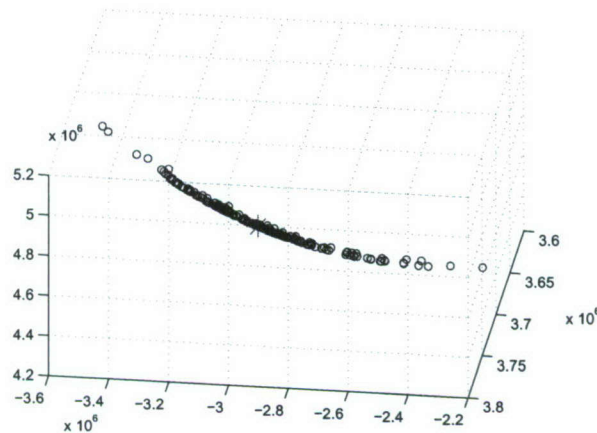

Figure 1: An example uncertainty region for a measurement. Each circle represents a possible measurement the radar would produce for a target located at the red star. From this angle, the full dimensions of the pancake cannot be seen. Only one of the two angle errors is apparent here. This figure is meant to demonstrate the thin range component, and curvature due to nonlinearity.

In these tests, the radar collects measurements in sine-space. This space is viewed in terms of a cartesian coordinate system aligned with the radar array face, and originating at the radar. The direction to the target is measured as the sine of the angle between the local target vector and the axes of the cartesian coordinate system. The radar only measures two sine components; these two components in combination with the target range suffice to compute the third component if desired. If a target is located at position

<div align="center">3</div>

(x, y, z) in the sensor cartesian system, then the full sensor measurement is:

$$r = \sqrt{x^2 + y^2 + z^2} + \nu_r$$
$$u = \frac{x}{r} + \nu_u$$
$$v = \frac{y}{r} + \nu_v,$$

where ν_r, ν_u, and ν_v are small independent errors that follow a Gaussian distribution with known covariance [5].

2.2   Dynamics Model

While some aspects of the force models relevant to orbit dynamics are well characterized (such as the gravitational field), others contain effects that may be difficult to model accurately. For example, low Earth orbit satellites may experience a highly uncertain drag force, both because our models of atmospheric density are inaccurate and because the drag coefficient of an object will vary with its orientation (and for many objects, particularly debris resulting from an event, it may not be possible to produce a drag coefficient estimate). Additionally, some elements of the full force model (such as force variations resulting from fluid and solid Earth tides) may not be represented in the orbit propagation model because inclusion of the full force model is computationally expensive. These *process noise* effects, representing errors in the motion models used for orbit propagation, will impact the total error in the predicted or retrodicted state of an object.

The dynamics of an orbiting object can be represented in a cartesian Earth-Centered Inertial (ECI) reference frame as

$$\dot{\mathbf{x}} = f(\mathbf{x}, t) = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ v_z \\ a_x(x, y, z, v_x, v_y, v_z) \\ a_y(x, y, z, v_x, v_y, v_z) \\ a_z(x, y, z, v_x, v_y, v_z) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \mu_x(t) \\ \mu_y(t) \\ \mu_z(t) \end{pmatrix} \tag{1}$$

where $\mu_x(t)$, $\mu_y(t)$, and $\mu_z(t)$ correspond to errors in the computed acceleration. We assume that the covariance of $\mu(t)$ is known from estimating the magnitude of the neglected terms in the force model.

For the purpose of initial orbit determination, the required fidelity of the model in use depends predominantly on the time diversity of the initial set of collected measurements. If all of the measurements to be used are collected in a relatively short window (such as a single pass of a low-Earth orbiting satellite), a lower-fidelity model may be more appropriate as fewer parameters must be estimated to produce a solution. For orbit propagation, we would prefer to use a sufficiently high-fidelity model that the dominant error in the result state is the propagation of the initial state error, and not accumulated model errors across the propagation time. In practice this is rarely achievable; the space

4

catalog encompasses a very large number of objects, and high-fidelity integrating propagators are too computationally expensive for most applications. Historically, a compromise has been reached between extremely low-fidelity models (Keplerian spherical-Earth or analytic J2 propagators) and high-fidelity integrators through the use of the Simplified General Perturbations Satellite Orbit Model 4 (SGP4).

In this study, however, we note that uncertainty generation and propagation rely somewhat less on the specifics of the model in use. To first order, the errors in the initial state estimate of an orbit are dominated by the measurement errors of the radar sensors we have considered. We are thus less interested in the model details and more in the correct representation of the *mismatch* between the simulator dynamics and the tracking model dynamics. For ease of prototyping, then, we have used the simplest possible dynamics formulation: that of two-body Keplerian dynamics on a spherical Earth. This dynamics model is briefly described below:

$$F_{\text{grav}} = G \frac{m_1 m_2}{r^3} \bar{r}, \tag{2}$$

where G is the universal gravitational constant, m_1 is the mass of the first object, m_2 is the mass of the second object, $\bar{r}$ is the difference in their positions, and r is the magnitude of that difference.

In the system of interest, this causes an acceleration of the small orbiting body equal to

$$a_{\text{grav}} = G \frac{M_E}{r^3} \bar{r}, \tag{3}$$

where M_E is the mass of the Earth, $\bar{r}$ is the difference in position between the center of the Earth and the object, and r is the distance between the two.

Note that none of the fundamental methods to be discussed rely on this model; the approach applies equally well to an arbitrary model selection. The only requirement for the use of an arbitrary model is that the process noise describing the difference between the model and the expected real object dynamics be sufficiently well characterized. We anticipate that, with some extensions, the SGP4 model could be augmented with uncertainty propagation at an acceptable computational cost.

3   Approach

In order to predict the target's location at some future point in time, we must first estimate its state at the times it is measured, and then determine its most likely path across the intervening time interval.

3.1   Initial Orbit Determination

The problem of determining the initial orbit is called "track initiation" in multiple target tracking language. Given a sequence of measurements $\{z_1, ..., z_n\}$ taken at $\{t_1, ... t_n\}$, respectively, the goal is to determine the position and velocity (or orbital elements) of the

orbiting target at some time, usually within the time spanned by the measurement times. Our goal is to solve the constrained nonlinear least squares optimization problem

$$\text{Minimize} \quad \sum_{k=1}^{n} \| z_k - h\left(\hat{\mathbf{x}}(t_k; \mathbf{y}, t_0)\right) \|_{R_k^{-1}}^2 \tag{4}$$

$$\text{Subject to} \quad \frac{d\hat{\mathbf{x}}}{dt} = f(\hat{\mathbf{x}}, t)$$

$$\hat{\mathbf{x}}(t_0) = \mathbf{y}.$$

Here, we are optimizing over the vector $\mathbf{y}$ given a time t_0, dyanmics $\frac{dx}{dt} = f(x, t)$ and the above measurement sequence. The dynamics may include a general gravity model and a host of other physical forces on the object such as drag and solar radiation pressure. In addition, one can include process noise in the above formulation as found in the books Jazwinski[4] and Sage and Melsa[9], or the paper by Poore [8].

There are a variety of optimization methods for solving the above problem, including Gauss-Newton, full Newton, or hybrid methods with a globalization procedure such as a line search or trust region method. (The Levenberg-Marquardt method may be viewed as a Gauss-Newton method with a trust region globalization.) To initialize the nonlinear least squares, one could filter and smooth the data or could use three measurements to approximate the position and velocity. The covariance of the estimate $\mathbf{y}$ is identified in the course of solving the nonlinear least squares problem ([9]).

3.2   Consider Analysis

Newton's method yields a covariance matrix based on the final solution and the statistically formulated objective function (4). As a result of both the nonlinear transformation between a radar measurement and a target position, and also the nonlinear dynamics of the system, the error in the estimate does not actually follow a Gaussian distribution even if the measurement errors are Gaussian. In Cartesian space, a covariance matrix can only represent an ellipsoidal uncertainty region. The curved pancake from figure 1 is flattened into a highly eccentric ellipsoidal region. This region does not accurately reflect the true system uncertainty. The two uncertainty regions are compared in figure 2.

One method of compensating for this approximation in the covariance computation is called a consider analysis. In a consider analysis, the covariances of the measurements are increased to also account for the error caused by the linearization of the nonlinear transformation. Here is a brief presentation. We assume the measurement equation to have the form $z = h(x, b) + \nu$ where $nu \in \mathcal{N}(0, R)$ is the measurement noise. Assuming one has obtained an estimate of the biases b, say $\hat{b}$, one can expand the measurement equation via $h(x, b) \approx h(x, \hat{b}) + D_b h(x, \hat{b})(\epsilon)$ where $\epsilon = b - \hat{b}$. The modified measurement equation then is

$$z = h(x, \hat{b}) + D_b h(x, \hat{b})(\epsilon) + \nu$$

Again, assuming ϵ and ν to be independent and ϵ has covariance C, the measurement error R_k in equation (4) is replaced by $R_k + D_b h(x, \hat{b}) C D_b^T h(x, \hat{b})$. The second term gives

6   Approved for public release;
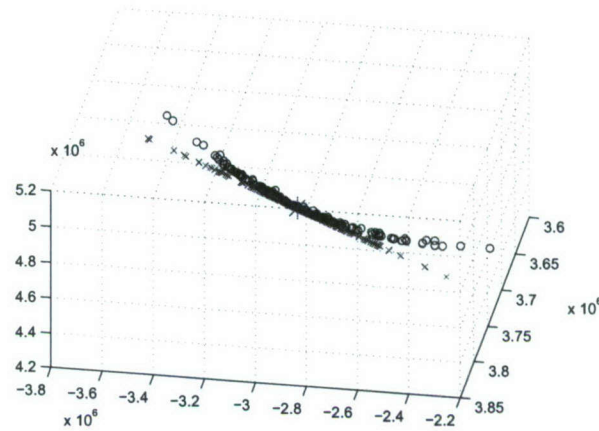distribution is unlimited.

Figure 2: The flattened uncertainty region caused by linearizing the transformation. The crosses are drawn from a linearized distribution with the same mean. The uncertainty region for this distribution does not capture the possible error of the measurements.

us the opportunity to account for the residual biases and for the accuracy achievable in the sometimes highly ill-conditioned transformation $h(x)$.

Said in another way, this procedure allows the covariance of the final solution to capture the truth, even in cases where the errors were actually larger than expected due to the transformation approximation. This method thickens the flat pancake so that the ellipsoidal uncertainty region captures the actual uncertainty. The effect is illustrated by the much thicker ellipsoidal region shown in figure 3.

The Schmidt-Kalman filter goes a step beyond the above consider analysis and builds the statistical cross correlation between the biases b and the state x and has been shown to be very effective for improving covariance consistency [7].

3.3   Propagating Uncertainty

The equations of motion (1) can be integrated (along with a covariance matrix representing the uncertainty of the state) using a Riccati equation to predict the position of the target at a future time. Direct integration of the equations of motion is used when a complete model of the orbital dynamics is desired, as it allows for a general acceleration estimate and error description to be inserted into the dynamics.

The Riccati equation for this problem linearizes the dynamics at each point in the integration. Linear propagation provides a very simple method of updating the covariances. The dynamics for the covariance become

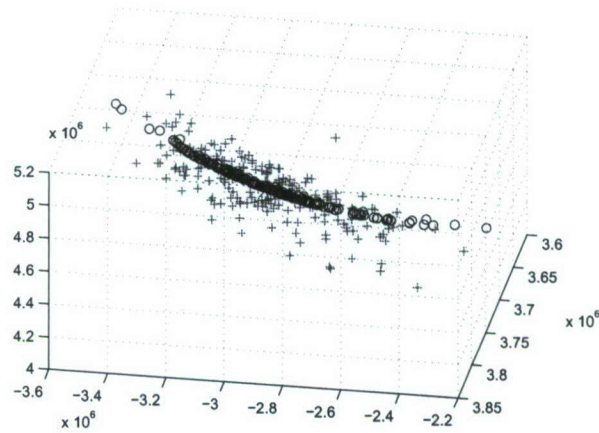$$\frac{dP}{dt} = F(\mathbf{x})P + PF(\mathbf{x})^T + Q(t),$$

Figure 3: The uncertainty region has been thickened to cover the actual errors, by using consider analysis. The uncertainty region represented by the plus symbols now contains the true errors.

where $F(\mathbf{x})$ is the Jacobian of the dynamics at the current state $\mathbf{x}$, and $Q(t)$ is the covariance associated with the random errors μ_x, μ_y, and μ_z.

An alternative to integration is to transform the cartesian position and velocity estimate into orbit elements. Orbit elements describe an elliptical orbit and the target's position along that orbit; variations on the classical Keplerian orbit elements can be used to describe a range of analytic solutions to the orbital dynamics problem. The simplest form of these is that used for a spherical Earth force model. These analytic methods are much more computationally efficient than an integrating propagator, at the expense of only being able to consider a subset of the forces on a space object. The transformation

from position, $\mathbf{r}$, and velocity, $\mathbf{v}$, into orbit elements is

$$r = \|\mathbf{r}\|, \ v = \|\mathbf{v}\|,$$
$$\mathbf{h} = \mathbf{r} \times \mathbf{v}, \ h = \|\mathbf{h}\|,$$
$$\mathbf{n} = \hat{\mathbf{k}} \times \mathbf{h}, \ n = \|\mathbf{n}\|,$$
$$\mathbf{e} = \frac{1}{\mu}\left(\left(v^2 - \frac{\mu}{r}\right)\mathbf{r} - (\mathbf{r}\cdot\mathbf{v})\mathbf{v}\right), \ e = \|\mathbf{e}\|,$$
$$a = \frac{\frac{h^2}{\mu}}{1 - e^2},$$
$$\cos(i) = \frac{\mathbf{h_k}}{h}, \ i < 180°$$
$$\cos(\Omega) = \frac{\mathbf{n_i}}{n}, \ \text{if } \mathbf{n_j} > 0 \text{ then } \Omega < 180°$$
$$\cos(\omega) = \frac{\mathbf{n}\cdot\mathbf{e}}{ne}, \ \text{if } \mathbf{e_k} > 0 \text{ then } \omega < 180°$$
$$\cos(\nu) = \frac{\mathbf{e}\cdot\mathbf{r}}{er}, \ \text{if } \mathbf{r}\cdot\mathbf{v} > 0 \text{ then } \nu < 180°,$$

where the orbit elements are a (semi-major axis of elliptical orbit), e (eccentricity of elliptical orbit), i (inclination angle of the orbit from the equatorial plane), Ω (angle of ascension through equatorial plane from the equinox), ω (angle between perigee and equatorial passage within the orbital plane), and ν (angle from perigee to current target position). For simple point mass models of gravity, only the current position along the orbit ν needs to be updated. The relationship between the time that has passed since the last time the target passed perigee and the angle ν is described by Kepler's equation [1].

$$\cos(E) = \frac{e + \cos(\nu)}{1 + e\cos(\nu)}$$
$$\Delta t = \sqrt{\frac{a^3}{\mu}}\left(E - e\sin(E)\right)$$

This second method of propagation does not inherently allow for appropriate propagation of uncertainty. However, because the propagation is extremely quick, a particle transform can be employed. In a particle transform, many pseudo-random values from an initial distribution are drawn, and each one is propagated the appropriate amount of time. If the dynamics model has random errors, small appropriate perturbations can be added to each particle during the propagation. After propagation the values represent the distribution at the later time. Figure 4 illustrates the results of the method.
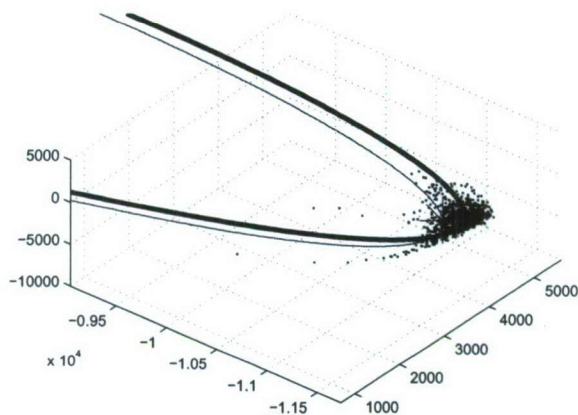
Figure 4: The thick line represents the mean orbit of the particles (black dots). The true orbit illustrated by the thin blue line lies within the cloud of particles. While there is error, the covariance of the particles in the cloud clearly captures the blue star where the target is truly located.

4 Simulation Results

4.1 Normalized Chi-Squared Metric

In order to test whether the covariance matrices developed using these approaches were appropriate, we examine covariance consistency. The mean squared error of an unbiased estimator that follows a Gaussian distribution with unit variance follows a chi-squared distribution with one degree of freedom. This distribution has an expected value of one. Furthermore, the mean squared error of an estimate divided by the variance of the estimate also follows this distribution. Analogously, the quantity

$$(\hat{x} - x^*)^T P_x^{-1} (\hat{x} - x^*), \tag{5}$$

where $\hat{x}$ is drawn from an m-dimensional Gaussian distribution with mean x^* and co-variance P_x^{-1} will have a chi-squared distribution with m degrees of freedom, which has a mean of m. We normalize by dividing out the dimension m and the mean once again equals one.

In order to determine if the covariance matrices developed using the methods described is an adequate representation of the errors in the estimates, we can examine the normalized chi-squared metric. If this value is far greater than one on average, then the covariance matrix is actually to small to capture the errors. If the value is less than one on average, then the covariance matrix is too large on average, and the actual errors are smaller than it indicates.

4.2   Scenario Description

A single target in orbit is produced following a Keplerian trajectory. It passes through the field of view of a radar platform that is sitting on the Earth. The radar generates measurements of a target by first transforming the true target position into its measurement space, and then adding independent Gaussian pseudo-random errors to these quantities. The covariances of these quantities are recorded along with the measurements.

We run the simulation in two different modes. In the first mode, no consider analysis is performed on the measurements; the measurements are passed directly into the batch estimator. In the second mode, a linearization compensation error covariance of 50 m^2 in each cartesian coordinate is added to the measurement covariance through the appropriate transformation. This additional uncertainty keeps the uncertainty region described by the covariance from being thin and flat, when it should be curved.

The nonlinear least squares problem (4) is solved to produce an estimate, $\mathbf{x}$, and a covariance matrix $P_{\mathbf{x}}$. The covariance consistency metric is computed for this estimate. The next step is to propagate the track states and their uncertainties for 12 hours. To do this we use the two different methods described: integrate the equations of motion using the Ricatti equations to carry the covariance, or use a particle transform into orbit elements, and perform the propagation in that space. In these simulations the true target trajectory was computed using a spherical Earth model, without any additional forces. Therefore, the process noise term $Q(t)$ was zero, and no random changes to the orbit element particles were added during propagation. The particle transformation used a sample size of 1000. After the propagation, the covariance consistency metric is computed, using an integrated spherical Earth motion model to produce the true state at the later time.

4.3   Data

The number of measurements, and the amount of time they span greatly affects the ability of the batch estimation process to correctly estimate the target state. With few measurements or a very short span of time, the estimator produces poor estimates. With many measurements spread over a long period of time, the estimator provides useful estimates. The driving force behind this phenomenon is the velocity portion of the original estimate. When only a few measurements are available, and they're all at nearly the same time, the measurement errors prevent an accurate estimate of the velocity from being obtained. If the measurements are widely spaced, or enough measurements are taken, this effect is mitigated.

The scenario described was run with between 5 and 40 measurements with average total span of between 4 and 195 seconds. Several variations of number of measurements and time spacing of measurements were simulated. Each variation was performed in 500 Monte Carlo runs in each mode (with or without consider analysis). The covariance consistency metric was averaged over these 500 runs.

After performing these experiments, we notice that the integration of the covariance

Table 1: Mean Normalized Chi-Squared Statistic from 500 MC Runs with Consider Analysis

Measurements	Spacing	Span	T_0 ECI χ^2	Integration χ^2	T_0 OE χ^2	Particle χ^2
5	1	4	2.5060	-	1.4423	1.3429
5	5	20	2.4973	-	1.1934	1.0300
5	10	40	2.4037	-	1.0513	1.0545
5	20	80	2.5733	-	1.1189	1.1941
5	40	160	1.7023	-	.70684	.79285
10	1	9	1.6321	-	1.3547	.93299
10	5	45	1.6691	-	.98633	.96807
10	10	90	1.5266	-	.92922	.98162
10	20	180	1.3378	-	.79309	.86993
20	1	19	1.2581	-	.97881	.92262
20	5	95	1.1174	-	.84290	.88579
20	10	190	1.0001	-	.76438	.86928
40	1	39	.96285	-	.85410	.80293
40	5	195	.82877	-	.70033	.76643

Table 2: Mean Normalized Chi-Squared Statistic from 500 MC Runs without Consider Analysis

Measurements	Spacing	Span	T_0 ECI χ^2	Integration χ^2	T_0 OE χ^2	Particle χ^2
5	1	4	4.3587	-	2.3315	2.1651
5	5	20	3.8840	-	1.4460	1.2805
5	10	40	3.0420	-	1.1065	1.1001
5	20	80	3.7265	-	1.1288	1.1843
5	40	160	2.8620	-	1.0555	1.0787
10	1	9	2.4654	-	1.7041	1.1698
10	5	45	2.2713	-	1.1022	1.1202
10	10	90	2.4220	-	1.2131	1.2611
10	20	180	2.0620	-	1.0976	1.1622
20	1	19	1.7244	-	1.0809	1.0095
20	5	95	1.5003	-	.99134	1.0131
20	10	190	1.3443	-	.98115	.97296
40	1	39	1.4096	-	.99437	1.0459
40	5	195	1.2076	-	1.0011	1.0612

using the Riccati equation cannot be performed in a stable manner with zero process noise. The integration quickly deteriorates the condition number of the matrix, until it is numerically singular, and no longer has a valuable meaning. Without the covariance matrix, the chi-squared consistency metric cannot be computed.

Based on tables 1 and 2, we can see that if enough measurements are available spread over a long enough time period, then the estimates represented as orbital elements can achieve consistent covariances, without the use of consider analysis. This is not the case for the ECI estimates, however. The elliptical uncertainty region represented by the ECI covariance matrix is the wrong shape. If it is to capture the true errors in a consistent manner, a consider analysis performed for the specific number of measurements, timing, and geometry must be performed. The additional uncertainty of 50 m^2 in each direction did make the scenario with 20 measurements taken one every 10 seconds consistent. However, this consider value was excessively large for the scenario with 40 widely spread measurements, and insufficient for the scenarios with only 10 measurements.

5   Conclusion

The goals of this project were twofold: (1) given a sequence of radar measurements, establish an object's position and velocity at a specified time and develop a consistent covariance matrix that represents the uncertainty in the initial state, and (2) determine a method of propagating this state and its uncertainty that maintains a consistent uncertainty measure for a period of hours or days.

We achieved the first of these goals by using Newton's method combined with a consider analysis as demonstrated in tables 1 and 2. One should the bias covariances arising from a bias estimator to better judge the effectiveness of the algorithm.

The second goal was attained by using a particle transform to describe the states in orbital element space. This is demonstrated in figure 4, as well as with the metrics in tables 1 and 2. We also attempted to use the Riccati equation in ECI, but this led to an unrealistic covariance procedure. Perhaps the use of the Riccati equation and then conversion back to orbital element space may be more appropriate.

In future work, a Schmidt-Kalman filter-smoother might be used instead of Newton's method to appropriately take into account the statistical cross correlation between biases and the state to be estimated. The inclusion of higher order gravity model and such forces as atmospheric drag and solar radiation pressure ([6]) in the dynamics. The appropriate distribution of perturbations to be added to particles would need to be researched. Finally, we would like to test these methods using different sensor models, such as optical telescopes.

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